JOURNAL OF AIRCRAFT
Vol. 42, No. 3, May–June 2005

Computationally Efficient Analysis and Optimization of Stiffened Thin-Walled Panels in Shear

Awie Viljoen* and A. G. Visser†
Council for Scientific and Industrial Research, Pretoria 0001, South Africa
and
Albert A. Groenwold‡
University of Pretoria, Pretoria 0002, South Africa

The computationally efficient analysis and optimum design of the buckling of stiffened, thin-walled shear panels in aircraft structures is discussed. Namely, the postbuckling behavior of these panels is assessed using the iterative procedure developed by Grisham. This procedure requires only linear finite element analyses, whereas convergence is typically achieved in as few as five iterations. An algorithm developed by (A. F. Grisham, "A Method for Including Post-Buckling of Plate Elements in the Internal Loads Analysis of Any Complex Structure Idealized Using Finite Element Analysis Methods," AIAA Paper 78-515, April 1978) using connect format, is compared with empirical methods of analysis frequently used in aircraft structures and also with a refined, nonlinear quasi-static finite element analysis. It is shown that the procedure proposed by Grisham overcomes some of the conservatism inherent in conventional methods of analysis. In addition, the method is notably less expensive than a complete nonlinear finite element analysis, which makes it attractive for use during initial design iterations, even though global collapse of a structure cannot be predicted. As an illustration of the optimal design of buckled, stiffened thin-walled structures, the Grisham algorithm is combined with a microgenetic algorithm. Important reductions in weight are obtained within relatively few function evaluations.

Nomenclature

A	=	area
E	=	Young's modulus
G	=	shear modulus
k	=	diagonal tension factor
L_x	=	length of web along the x axis
L_y	=	length of web along the y axis
N	=	internal loads of structure
t	=	thickness of web
α	=	diagonal tension angle
γ_{xy}	=	web shear distortion
γ_{xy_c}	=	web shear distortion component due to compressive buckling
$\gamma_{xy_{DT}}$	=	Postbuckled shear distortion component of web
ϵ_x, ϵ_y	=	web normal strain
$\epsilon_{x_c}, \epsilon_{y_c}$	=	web compressive buckling strain
$\epsilon_{x_{DT}}, \epsilon_{y_{DT}}$	=	web diagonal tension strain
μ	=	poisson's ratio
σ_x, σ_y	=	web normal stress
$\sigma_{x_c}, \sigma_{y_c}$	=	web compressive buckling stress
$\sigma_{x_{cr}}, \sigma_{y_{cr}}$	=	web modified critical normal buckling stress
$\sigma_{x_{DT}}, \sigma_{y_{DT}}$	=	web diagonal tension stress
τ_{cr}	=	web critical buckling shear stress
τ_{xy}	=	web shear stress
$\tau_{xy_{cr}}$	=	web modified critical buckling shear stress

Received 24 November 2003; revision received 9 March 2004; accepted for publication 9 March 2004.

*Engineer, Division of Manufacturing and Materials Technology, Pretoria 0001, South Africa.

†Technical Specialist, Division of Manufacturing and Materials Technology.

‡Professor, Department of Mechanical Engineering, University of Pretoria.

I. Introduction

IN many aircraft structures, thin sheet structural components or panels are designed to buckle under shear load. On buckling, the internal loads and stresses in neighboring panels and the surrounding structure can change significantly. Hence, detailed analysis of the effects of buckling are important.

In the early days of aircraft design, postbuckling effects were taken into account through the theory of pure diagonal tension (PDT) proposed by Wagner.[1,2] However, in practice PDT proved to be very conservative. Wagner's approach was gradually modified to eventually become the more general approach of incomplete diagonal tension (IDT). The IDT approach was developed by NACA in the 1950s, after conducting an extensive testing program to generate empirical relations. This approach, also known as the NACA method, later became an accepted design approach used by many aircraft manufacturers, even though the theory is still considered conservative.

One of the factors neglected in the NACA approach is the interaction of stresses in each panel element on the element allowables, namely, the combination of compression and shear buckling, diagonal tension, and postbuckled skin softening in shear. As an alternative to IDT, nonlinear finite element codes can be used to assess buckling, even though design-by-rule failure criteria are more difficult to assess.[3] In addition, nonlinear finite element analyses are computationally very expensive during initial design iterations.

In 1978, Grisham[4] proposed an iterative procedure for the analysis of postbuckling behavior of thin-walled shear panels as used in aircraft structures. This procedure requires only linear finite element analyses, whereas convergence is typically achieved in as few as five iterations. Grisham's algorithm is attractive in optimization and during initial design iterations because the computational effort required is relatively low. Some of the salient features of the algorithm are as follows:

1) Convergence is usually achieved rapidly. Typically, as few as five iterations are required to obtain convergence to within 2% variation between successive values of diagonal tension $\sigma_{x_{DT}}$ and $\sigma_{y_{DT}}$.

2) Provision is made for compressive buckling in both the length and width directions of panels, as well as shear buckling. The latter causes the development of diagonal tension, accompanied by

associated loading of the surrounding structure and softening of the buckled plate in shear.

3) The interaction of compression–compression and shear buckling is accounted for.

4) The stiffness matrix of the finite element model is not altered in any way to include post-buckling effects.

5) Because the self-equilibrated prestrains do not modify the stiffness matrix, precisely equilibrated and compatible solutions are obtained.

6) The prestrains calculated give a direct indication of the degree of softening of the structure caused by buckling.

In this paper, the iterative algorithm developed by Grisham[4] is implemented to assess and evaluate the onset and magnitude of buckling in thin-walled flat panels. For the sake of verification, the results obtained using Grisham's algorithm are compared with an example taken from the literature, as well as with a refined nonlinear finite element analysis. It is intended that the computational efficient method proposed by Grisham be used during initial design iterations, as well as during computational demanding structural optimization iterations. An initial investigation into optimization then offered. In doing so, the Grisham algorithm is combined with a micro genetic algorithm. Allowable stresses are accommodated using a simple penalty formulation. It is demonstrated that few function evaluations suffice in improving initial designs.

II. Grisham Algorithm

For the sake of clarity, the iterative algorithm of Grisham is only briefly outlined and summarized here; for details, the reader is referred to the original work by Grisham.[4]

1) Calculate the internal loads N_i of the structure under consideration, using a linear finite element analysis. The stiffness matrix of the linear finite element analysis is calculated once only and retained for following iterations because small displacement theory applies.

2) Evaluate the onset of buckling using the internal loads obtained in step 1 based on analytical plate buckling criteria and an interaction equation.

3) If buckling does not occur, stop. Otherwise, calculate the post-buckling relaxation of plates under compressive load σ_{x_c} and σ_{y_c}, the post-buckled shear distortion $\gamma_{xy_{\mathrm{DT}}}$, and the associated diagonal tension $\sigma_{x_{\mathrm{DT}}}$ and $\sigma_{y_{\mathrm{DT}}}$.

4) Calculate the postbuckling strains $\triangle\epsilon_x$ and $\triangle\epsilon_y$ to relieve the compression and shear load that exceed the plate capability.

5) The postbuckling strains calculated now become the prestrains for the next iteration.

6) If the ratio of the final compression stress σ_{x_c} and σ_{y_c} in the buckled plate to the modified critical buckling stress $\sigma_{x_{\mathrm{cr}}}$ and $\sigma_{y_{\mathrm{cr}}}$ approaches unity and the diagonal tension stress $\sigma_{x_{\mathrm{DT}}}$ and $\sigma_{y_{\mathrm{DT}}}$ converges, stop. Otherwise, go to step 1.

III. Verification of the Grisham Algorithm

To verify the results of our implementation of Grisham's algorithm, we consider the simple structure with six panels schematically shown in Fig. 1a. Detailed geometric and loading detail may be found in Ref. 5. The panel width-to-depth ratios are 1:2.85, and the web material is 2024-T3 alloy sheet with a thickness of 0.635 mm. The two T-shaped flanges are fabricated from 7075-T6 alloy extrusions and the angled uprights from 2014-T6 alloy.

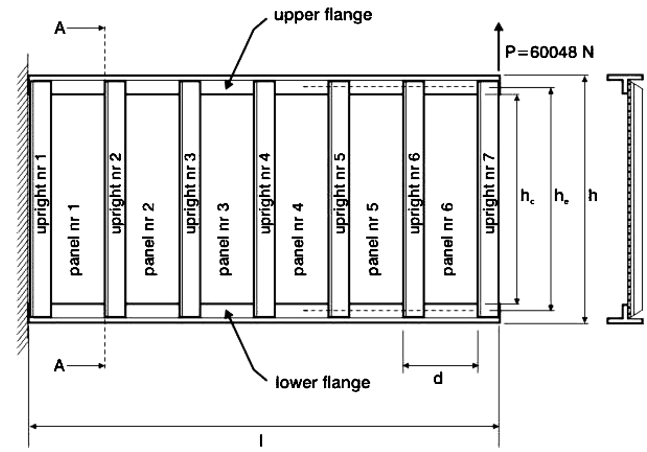
Two different methods are used to solve the problem, namely, the modified Wagner equations based on Refs. 1 and 2 and the NACA approach.[6,7] The results of both these approaches are then compared to the current implementation of the Grisham[4] algorithm. Numerical results are presented in Table 1. In Table 1, the results for panels 1 and 6 are ignored because they include edge effects that are not accounted for in the NACA or Wagner approaches. Only five iterations are required for the Grisham results; this means that the system equations are solved only five times, while the stiffness matrix remains constant throughout.

The results for the four panels obtained with the Grisham algorithm are very similar and compare closely to those of the other two methods. The value of the diagonal tension factor k obtained with the Grisham algorithm is slightly lower than that of the NACA method. The Grisham algorithm takes into account compression–compression buckling in both directions through an interaction equation that has an effect on the magnitude of the diagonal tension. In the NACA method, this is not taken into account, resulting in conservative predictions. (When the effect of compression–compression buckling is neglected in Grisham's method, the NACA result of $k = 0.69$ is obtained exactly.) The foregoing results are quite accurate, notwithstanding the coarse finite element mesh. Little is to be gained by mesh refinement (not shown).

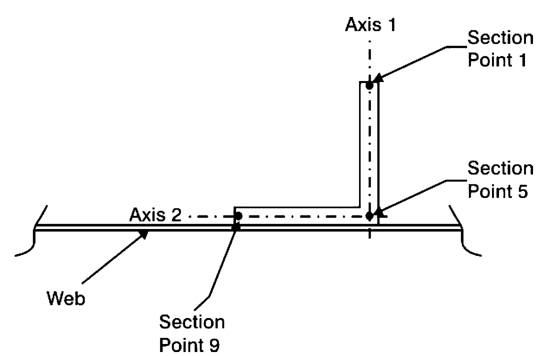
The critical shear stress τ_{cr} (the shear stress at the onset of buckling) for the Grisham algorithm and Wagner approach are based on analytical relations that depend on geometry and material properties. The NACA critical shear stress τ_{cr} includes empirical data in its calculation, relating to the stiffness of the flanges and uprights. The large difference between the critical shear stress value τ_{cr} and the total web shear stress shows that the final load carried by the panel is almost 50 times higher than initial buckling shear stress. The low value of τ_{cr} is a result of the relatively thin plate ($t = 0.635$ mm).

Table 1 Comparison between Grisham algorithm, modified Wagner equations, and NACA approach

	Grisham algorithm					
Variable	Panel 2	Panel 3	Panel 4	Panel 5	Modified Wagner	NACA
k	0.649	0.673	0.673	0.635	——	0.690
α, degrees	42.1	42.0	42.0	42.2	43.0	41.3
τ_{xy}, MPa	126.9	127.3	127.2	127.3	130.4	135.1
τ_{cr}, MPa	2.624	2.624	2.624	2.624	2.358	2.551



a) Geometry



b) Definition of the section points

Fig. 1 Verification example: cantilever beam found in Ref. 5.

IV. Comparison with a Nonlinear Finite Element Analysis

In this section, the Grisham algorithm results for the verification problem obtained in the preceding section are now compared with the results of a complete nonlinear finite element analysis. Global load control methods, for example, the arc-length control method

a) Onset of shear buckling, after 13 load increments

b) Excessive shear buckling at the end of the analysis, after 102 load increments

Fig. 2 Verification example: development of shear buckling with the nonlinear finite element analysis.

proposed by Riks[8] and Wempner,[9] are suitable for global buckling and postbuckling analysis. However, they are not ideal when the buckling is localized (when there is a local transfer of strain energy from one part of the model to neighboring parts). Alternatives are to analyze the problem dynamically, or to introduce an artificial damping factor. In the dynamic case, the strain energy released during local buckling is transformed into kinetic energy; in the damping case the strain energy is dissipated. To solve a quasi-static problem dynamically is expensive, however. Hence, the modified Riks[8] algorithm, combined with damping, is selected as an analysis option in the ABAQUS® environment.

The onset of shear buckling occurs at an applied load of 1552 N, resulting in a nominal shear stress of $\tau_{\mathrm{cr}} = 3.370$ MPa, which is slightly higher than the values predicted by the Grisham algorithm ($\tau_{\mathrm{cr}} = 2.624$ MPa), the Wagner method ($\tau_{\mathrm{cr}} = 2.358$ MPa) and the NACA approach ($\tau_{\mathrm{cr}} = 2.551$ MPa). The nonlinear finite element method takes the torsional stiffness of the flanges and uprights into account and, therefore, gives a more accurate, higher critical stress value. A less conservative analytical estimate by Fehrenbach[10] gives a value of $\tau_{\mathrm{cr}} = 3.195$ MPa for this problem.

The out-of-plane displacement results predicted by the nonlinear finite element analysis are shown in Figs. 2a and 2b. The deflection of the structure is, of course, another variable that can be used for comparative and validation purposes. These are as follows. The bottom node of the loaded end deflects 22.3 mm as predicted by the Grisham algorithm and 25.05 mm as predicted by the nonlinear finite element analysis.

A run-time comparison between the Grisham algorithm and the nonlinear finite element analysis indicates the high efficiency of the Grisham algorithm from a computational point of view. To do this effectively, identical mesh discretizations are used for the two models. The results are shown in Table 2, where the mesh given represents the mesh for each panel. All analyses were run on a Hewlett-Packard C200 workstation. It was impossible to ensure that the machine was not loaded by other users; hence, the results are of qualitative value only. Nevertheless, the Grisham algorithm is notably more efficient. In addition, the meshes used for the Grisham algorithm may in practice be much less refined than those required for the nonlinear finite element model (FEM).

Table 2 Total wall-clock times for the Grisham method and nonlinear finite element analysis as a function of mesh discretization

	Time	
Mesh	Grisham algorithm	Nonlinear FEM
3 × 3	31 s	7 min, 42 s
10 × 30	30 min, 39 s	4 h, 59 min

V. Structural Optimization

Grisham's algorithm is attractive for use in optimization because the computational effort required per function evaluation is relatively low. The algorithm can easily be combined with a well known but simple optimization algorithm, namely, the genetic algorithm (GA). In this section, the example considered in the preceding sections is, therefore optimized with respect to minimum mass, using a micro (μ) GA (μ-GA) with a binary representation, (for example, as by Carroll.[11] For simplicity, continuous design variables are used. Although a GA is not necessarily the best algorithm for the problem under consideration, it is selected here for ease in using discrete design variables in future.

The development of this section is as follows: First, the general optimal design problem is formulated. Next, the GA is briefly outlined, where after the differences between a GA and a μ-GA are outlined. (A μ-GA is based on and is derived from the GA.) The section then concludes with optimal design results for the example problem.

A. Objective Function and Constraints

The optimal design problem under consideration is formulated as follows. Find the minimum f^* such that

$$f^* = f(\boldsymbol{x}^*) = \text{minimum } f(\boldsymbol{x}) \tag{1}$$

subject to the general inequality constraints

$$g_j(\boldsymbol{x}^*) \leq 0, \qquad j = 1, 2, \ldots, m \tag{2}$$

where $\boldsymbol{x}$ is a column vector in $\mathbb{R}^n$ and f and g_j are scalar functions of the design variables $\boldsymbol{x}$. Here $\boldsymbol{x}$ is subject to the subsidiary conditions $x_i^l \leq x_i \leq x_i^u$, with x_i^l and x_i^u, respectively, representing prescribed lower and upper bounds on x_i. Equation (1) is the objective function. In our case, f represents the mass of the structure, whereas Eq. (2) is expressed in terms of allowable stress.

B. GA

GAs[12] are stochastic implicit enumeration methods based on Darwinism and, in particular, on the natural theory of survival of the fittest. In brief, GAs attempt to improve the fitness of designs (expressed in terms of a scalar objective function) in consecutive generations. The initial generation is populated in a random fashion with chromosomes representing possible discrete designs. The genetic operators of selection, crossover, and mutation are then used in a controlled random manner to ensure that fit parents have a high probability of passing fit genetic material to their offspring.

Even though GAs are infamous for high computational costs (a large number of function evaluations), GAs are attractive to engineers, because their construction is quite simple. Numerous applications of GAs in engineering optimization have been presented. In addition, GAs are also suitable for implementation on massively parallel processing machines and lend themselves to be tailored according to the behavior of the objective function under consideration.[13]

The most important aspects of a GA are briefly described in the following subsections.[14]

1. *Representation of Design Variables*

We consider an initial design population, constituting of e design vectors (or strings in GA jargon) $\boldsymbol{x}$, created by a random selection of the variables in the variable space for each design. The values of the variables in the strings must be represented by a unique coding scheme. We opt for binary coding, which is powerful, frequently used, and above all simple.

2. Selection

The selection operation selects e strings from the current population to form the mating pool. The strings corresponding to fit objective function values have the greatest chance to be selected for mating and, hence, to contribute to future generations. Although a large number of different selection processes are possible, we discuss only tournament selection.

Tournament selection simulates the process in which individuals compete for mating rights in the population.[11] In the GA, e tournaments are held between a subgroup of strings chosen randomly from the existing population. The design from each tournament with the lowest function value is selected for the mating pool.

3. Crossover

After selecting e strings for the mating pool, new designs are explored by the crossover process. Crossover allows selected individuals to trade characteristics of their designs by exchanging parts of their strings. The mating pool strings are randomly grouped into pairs, and a breaking point in the strings for each pair is chosen randomly. The values at the string positions after the breaking point are interchanged between the pair and the new designs are copied to the new generation. Crossover for each pair is applied with a given probability p_c, usually between 0.6 and 1.

4. Mutation

The mutation operation protects against complete loss of genetic diversity by randomly changing bit values in a string. For each bit in the population, a random number is generated, and the bit value is changed if the random number is less than the prescribed probability of mutation p_m. Mutation typically occurs at low probability, else convergence can be impaired.

C. μ-GA

A μ-GA is an implementation of the GA in which rebirth replaces mutation. The basic idea is to use very small populations, for example, five individuals. It is then hoped that the small population converges very quickly, where after one or at most two individuals are retained using tournament selection. The remaining individuals are then killed off and randomly repopulated, a process called rebirth.

Rebirth was first proposed by Galante.[15] The implementation of Carroll[11] is similar to that of Galante,[15] except that Galante used larger generations than the norm in the μ-GA. In addition, Galante retained mutation, which probably is superfluous.

Although the "no-free-lunch" theorems[16,17] effectively prohibit a general, exhaustive comparison between the GA and the μ-GA, the latter has a secondary advantage. A reasonable solution may sometimes be obtained relatively quickly, due to the small generations typically employed. This may be desirable when evaluation of the objective function is computationally expensive, for example, when using nonlinear finite element analyses, and when a visual inspection of the current best solution can be used to defend additional computational effort.

An inherent drawback of a μ-GA is that premature convergence is likely. Criteria for terminating a population are obvious: It can, for example, be done when a prescribed fraction, for example, 0.8, of the population has converged to a given value. However, knowing when enough restarts have been made is not simple and will probably be influenced by factors such as computational effort and the quality of the solution found to date.

D. Optimization of the Verification Example

The minimum-mass design of the stiffened shear webs is taken as the objective function, whereas the design variables represent the dimensions of the six panels. The single constraint g considered in the optimization phase is the maximum allowable prescribed postbuckled stress expressed as

$$g = \tau_{xy}^{\max} - \tau_{xy}^{\text{pres}} \tag{3}$$

where τ_{xy}^{pres} is the prescribed, maximum allowable postbuckled stress. The objective function is then formulated as

$$f = \sum_{k=1}^{l} m_k + \mu\rho(g)^2 \quad \rho > 0 \text{ and prescribed} \tag{4}$$

where l is the number of structural members, m the mass of member k, and

$$\mu = \begin{cases} 0 & \text{if } g \le 0 \\ 1 & \text{otherwise} \end{cases} \tag{5}$$

Furthermore, a large number of bound constraints are included to the single constraint on stress to ensure validity of the solution. These constraints are all expressed in terms of the relationships given by Grisham's method[4] and are included in Tables 3 and 4, which give the numerical results.

The example is optimized using two different sets of design variables. First, 4 geometric variables are selected, where after 11 geometric variables are used. With reference to Fig. 1a, the design variables for case 1 are the upper flange area, the lower flange area, the upright area (all equal), and the web thickness (all equal). For case 2, the variables are the upper flange area, the lower flange area, the area of uprights 2–6 (all different), and the web thickness of panels 2–5 (all different). In both cases, panels 1 and 6 are assumed to be of prescribed geometry.

1. μ-GA Parameters

In the implementation of the μ-GA, we use a small population size of five and a uniform crossover with an 80% probability. One

Table 3 Case 1, optimum results using μ-GA

Variable	Description	Initial value	Final value	Bounds
x_1	Lower flange area, mm^2	243.87	201.0	$100 \le x_1 \le 250$
x_2	Upper flange area, mm^2	435.48	400.4	$200 \le x_2 \le 450$
x_3	Upright area, mm^2	151.21	150.4	$50 \le x_3 \le 170$
x_4	Web thickness, mm	0.635	0.5188	$0.3 \le x_4 \le 0.9$

Table 4 Case 2, optimum results using μ-GA

Variable	Description	Initial value	Final value	Bounds
x_1	Lower flange area, mm^2	243.9	197.7	$100 \le x_1 \le 250$
x_2	Upper flange area, mm^2	435.5	396.5	$200 \le x_2 \le 450$
x_3	Area of upright 2, mm^2	151.2	140.1	$50 \le x_3 \le 170$
x_4	Area of upright 3, mm^2	151.2	142.8	$50 \le x_4 \le 170$
x_5	Area of upright 4, mm^2	151.2	138.5	$50 \le x_5 \le 170$
x_6	Area of upright 5, mm^2	151.2	141.1	$50 \le x_6 \le 170$
x_7	Area of upright 6, mm^2	151.2	141.9	$50 \le x_7 \le 170$
x_8	Panel 2, web thickness, mm	0.635	0.506	$0.3 \le x_8 \le 0.9$
x_9	Panel 3, web thickness, mm	0.635	0.494	$0.3 \le x_9 \le 0.9$
x_{10}	Panel 4, web thickness, mm	0.635	0.490	$0.3 \le x_{10} \le 0.9$
x_{11}	Panel 5, web thickness, mm	0.635	0.497	$0.3 \le x_{11} \le 0.9$

child per pair of parents is produced, and parent selection is based on tournament selection. Elitism is included.

Eccentricity of the uprights is taken into account. The cross sections in the example are used: T sections for the flanges and angle sections for the uprights. During the optimization phase, the finite element mesh is never updated or changed in any way from the initial model. Because we are merely interested in a demonstration of capability, rather than finding very good optima for the pathological problem under consideration, the μ-GA is terminated after only 20 generations, namely, 100 function evaluations.

2. *Optimal results*

For case 1, using the μ-GA, the optimum mass is obtained as 5.52 kg after 37 generations. This gives an 11% saving on mass from the original mass of 6.12 kg, which is significant in aircraft structures. At the optimum, the stresses are $\sigma_{\text{mises}} = 325.8$ MPa in the web and $\sigma_{\text{mises}} = 392.4$ MPa in the flanges. For case 2, the optimum mass is obtained as 5.366 kg after 23 generations. This gives a 14.08% saving on mass from the original mass of 6.122 kg. This is very similar to the four-variable case. At the optimum, the stresses are $\sigma_{\text{mises}} = 307.7$ MPa in the web and $\sigma_{\text{mises}} = 427.8$ MPa in the flanges. Note that neither case 1 nor case 2 are converged when the algorithm is terminated because neither the stress constraint nor the variable bounds are active in either case. Case 2 does produce a more optimal result. However, the objective of demonstrating an effective reduction in weight in only 20 generations is achieved quite easily.

VI. Discussion

A. Evaluation of the Grisham Algorithm: Comparison with Wagner, Modified Wagner, and NACA Approaches

For the verification example studied, the Grisham algorithm gives comparable results to those obtained using the Wagner and NACA approaches, with the Grisham algorithm being less conservative. The web results compared very well. The diagonal tension factor k is calculated to be between 2.5 and 6.0% lower (depending on the panel studied). The diagonal tension angle α of 42.2 deg is roughly halfway between the Wagner and NACA predictions, with the total shear stress in the web being 6% lower than that predicted by NACA.

B. Comparison with Full Nonlinear Analysis

Although no diagonal tension factor is obtained from the nonlinear finite element analysis, a qualitative inspection reveals that the direction of the principle stresses in the nonlinear analysis agree well with the diagonal tension angle calculated using the Grisham algorithm.

The web critical buckling shear stress τ_{cr} calculated with the nonlinear finite element analysis is slightly higher and less conservative than that of the other methods (Wagner, NACA, and Grisham). The agreement between the nonlinear analysis and the Grisham web stress results are within 22%. The deflection results compare reasonably well, with the Grisham algorithm tip deflection being 11% lower than that of the nonlinear finite element analysis.

From a computational effort point of view, the Grisham algorithm is superior by far. Not only is the structure solved using linear elastostatic theory, but effective mesh sizes may also be far more coarse than meshes used in a nonlinear analysis. Hence, the algorithm can very effectively be combined with a μ-GA to obtain designs of increased efficiency during initial design iterations.

VII. Conclusions

We have implemented the iterative procedure proposed by Grisham algorithm to evaluate the buckling of stiffened, thin-walled shear panels in aircraft structures. Comparisons with empirical methods of analysis frequently used in aircraft structures, as well as a refined, nonlinear quasi-static finite element analysis, confirm the suitability of the procedure during initial design iterations, as well as during optimization.

The Grisham algorithm requires only linear finite element analyses and converges very quickly, making it computationally efficient. In addition, both compressive and shear buckling, and the interaction between these, are provided for.

Much of the inherent conservatism associated with conventional methods of analysis is overcome with Grisham's method, while the implementation is simpler than a nonlinear finite element analysis.

References

[1] Wagner, H., "Ebene Blechwandträger mit sehr dünnen Stegbleich," *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Vol. 20, 1929.

[2] Wagner, H., "Flat Sheet Metal Girders with Very Thin Webs," Technical Rept. TM 604, 605 and 606, NACA, Feb. 1931.

[3] Mello, R. M., Sherrer, R. E., and Musgrove, M. D., "Intermediate Diagonal Tension Field Shear Beam Development for the Boeing SST," *Journal of Aircraft*, Vol. 9, 1972, pp. 470–476.

[4] Grisham, A. F., "A Method for Including Post-buckling of Plate Elements in the Internal Loads Analysis of Any Complex Structure Idealized Using Finite Element Analysis Methods," AIAA Paper 78–515, April 1978.

[5] Bruhn, E. F., *Analysis and Design of Flight Vehicle Structures*, Tri-State Offset Co., Cincinnati, OH, 1973, pp. C11.1–C11.28.

[6] Kuhn, P., Peterson, J. P., and Levin, L. R., "A Summary of Diagonal Tension. Part 1: Methods of Analysis," Technical Rept. TN 2661, NACA, May 1952.

[7] Kuhn, P., Peterson, J. P., and Levin, L. R., "A Summary of Diagonal Tension. Part 2: Experimental Evidence," Technical Rept. TN 2662, NACA, May 1952.

[8] Riks, E., "The Application of Newton's Method to the Problem of Elastic Stability," *Journal of Applied Mechanics*, Vol. 39, 1972, pp. 1060–1066.

[9] Wempner, G. A., "Discrete Approximations Related to Nonlinear Theories in Solids," *International Journal of Solids and Structures*, Vol. 7, 1971, pp. 1581–1599.

[10] Fehrenbach, J. M., "Elements pour le calcul des structures d'avions," *Ecole Nationale Superieure de l'Aeronautique et de l'Espace*, 1974, pp. 16, 17.

[11] Carroll, D. L., "Chemical Laser Modeling with Genetic Algorithms," *AIAA Journal*, Vol. 34, No. 2, 1996, pp. 338–346.

[12] Goldberg, D. E., *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison–Wesley, Reading, MA, 1989.

[13] Groenwold, A. A., Stander, N., and Snyman, J. A., "A Regional Genetic Algorithm for the Discrete Optimal Design of Truss Structures," *International Journal for Numerical Methods in Engineering*, Vol. 44, No. 6, 1999, pp. 749–766.

[14] Bolton, H. P. J., "Parallel Competing Algorithms in Global Optimization," M.S. Thesis, Dept. of Mechanical Engineering, University of Pretoria, Pretoria, South Africa, Nov. 2000.

[15] Galante, M., "Genetic Algorithms as an Approach to Optimize Real-World Trusses," *International Journal for Numerical Methods in Engineering*, Vol. 39, No. 3, 1996, pp. 361–382.

[16] Wolpert, D., and Macready, W., "No Free Lunch Theorems for Optimization," *IEEE Transactions on Evolutionary Computing*, Vol. 1, No. 1, April 1997, pp. 67–82.

[17] English, T. M., "Evaluation of Evolutionary and Genetic Operators: No Free Lunch," *Evolutionary Programming V: Proceedings Fifth Annual Conference on Evolutionary Programming*, edited by L. J. Fogel, P. J. Angeline, and T. Baeck, MIT Press, Cambridge, MA, 1995, pp. 163–169.